PHYSICS

1. The dimensions of mobility of charge carriers are:

A) $\left[M^{-2}T^{2}A\right]$ B) $\left[M^{-1}T^{2}A\right]$ C) $\left[M^{-2}T^{3}A\right]$ D) $\left[M^{-1}T^{3}A\right]$ E) $\left[M^{-1}T^{2}A^{-1}\right]$

2. A coin is dropped in a lift. It takes time t_1 to reach the floor when lift is stationary. It takes t_2 when lift is moving up with constant acceleration, then :

A) $t_1 = t_2$ B) $t_1 > t_2$ C) $t_2 > t_1$ D) $t_1 >> t_2$ E) $t_2 >> t_1$

3. If the angle between two vectors $\vec{A}$ and $\vec{B}$ -is 120°, its resultant C will be:

A) $C = \left|\vec{A} - \vec{B}\right|$ B) $C < \left|\vec{A} - \vec{B}\right|$ C) $C > \left|\vec{A} - \vec{B}\right|$ D) $C = \left|\vec{A} + \vec{B}\right|$ E) $C > \left|\vec{A} + \vec{B}\right|$

4. A projectile is given an initial velocity of $\hat{i} + 2\hat{j}$. The cartesian equation of its path is: $\left(g = 10\ m/s^2\right)$

A) $y = 2x - 5x^2$ B) $y = x - 5x^2$ C) $4y = 2x - 5x^2$ D) $y = 2x - 25x^2$ E) $y = x - 25x^2$

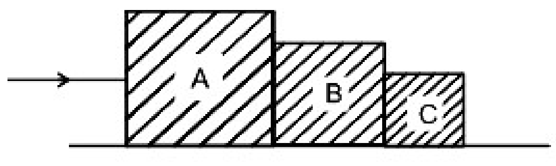
5. Three forces are acting on a particle of mass initially in equilibrium. If the first 2 forces (R_1 and R_2) are perpendicular to each other and suddenly the third force (R_3) is removed, then the acceleration of the particle is:

A) $\frac{R_3}{m}$ B) $\frac{R_1 + R_2}{m}$ C) $\frac{R_1 - R_2}{m}$ D) $\frac{R_1}{m}$ E) $\frac{R_2}{m}$

6. A heavy uniform chain lies on a horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain, that can hang over one edge of the table is:

A) 10% B) 25% C) 35% D) 15% E) 20%

7. Three blocks A, B and C of masses 4 kg, 2 kg and 1 kg respectively, are in contact on a frictionless surface as shown in figure. If a force of 14 N is applied on the 4 kg block, then the contact force between A and B, is:



A) 6 N B) 8 N C) 18 N D) 2 N E) 10 N

8. A bullet of mass m moving with velocity υ strikes a suspended wooden block of mass M. If the block rises to a height h, the initial velocity of the bullet will be:

A) $\sqrt{2gh}$ B) $\frac{(M+m)}{m}\sqrt{2gh}$ C) $\frac{m}{(M+m)}\sqrt{2gh}$

D) $\frac{(M-m)}{m}\sqrt{2gh}$ E) $\frac{m}{(M-m)}\sqrt{2gh}$

9. The angular momentum of a particle describing uniform circular motion is L. If its kinetic energy is halved and angular velocity doubled, its new angular momentum is :

A) 4L B) $\frac{L}{8}$ C) $\frac{L}{2}$ D) 2L E) $\frac{L}{4}$

10. Two circular discs are of the same thicknesws. The diameter of A is twice that of B. The moment of inertia of A as compared to that of B is:

A) twice as large B) four times as large C) 8 times as large
D) 16 times as large E) can predict

11. A satellite is orbiting around the earth. By what percentage should we increases its velocity so as to enable it to escape away from the earth?

A) 41.4% B) 50% C) 82.8% D) 100% E) 62.5%

12. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at a height 3L/4 from its lower end is:

A) W_1 / S B) $\left(W_1 + \frac{W}{4}\right)S$ C) $\left(W_1 + \frac{3W}{4}\right)/S$ D) $(W_1 + W)S$ E) $(W_1 + 3W)S$

13. The excess of pressure inside the first soap bubble is three times that inside the second bubble. The ratio of volume of the first to that of the second bubble is:

A) $1:3$ B) $1:9$ C) $27:1$ D) $9:1$ E) $1:27$

14. The wettability of a surface by a liquid depends primarily on:

A) surface tension B) density
C) angle of contact between the surface and the liquid D) viscosity E) excess pressure

15. A wooden badll of density D is immersed in water of density d to a depth h below the surface of water and then released. Upto what height will the ball jump out of water?

A) $\frac{d}{D}h$ B) $\left(\frac{d}{D} - 1\right)h$ C) h D) zero E) $(d - D)h$

16. Stream-line flow is more likely for liquids with:

A) low density and low viscosity B) high viscosity and high density
C) high viscosity and low density D) low viscosity and high density
E) equal density and viscosity

17. The temperature at which the reading of a Fahrenheit thermometer will be double that of centigrade thermometer is:

A) 160° B) 180° C) 32° D) 100° E) 240°

18. Two spheres A and B have diameters in the ratio 1:2, densities in the ratio 2:1 and specific heats in the ratio 1:3; find the ratio of their thermal capacities:

A) $1:6$ B) $1:9$ C) $1:3$ D) $1:4$ E) $1:12$

19. A body cools from a temperature 3T to 2T in 10 minutes. The room temperature is T. Assume that Newton's law of cooling is applicable. The temperaturef of the body at the end of next 10 minutes will be:

A) T B) $\frac{7T}{4}$ C) $\frac{3T}{2}$ D) $\frac{4T}{3}$ E) $\frac{6T}{2}$

20. During an adiabatic process, the pressure of a gas is proportional to the cube of its absolute temperature. The value of C_p/C_v for the gas is:

A) 3/5 B) 4/3 C) 5/3 D) 3/2 E) 3/4

21. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the string of the pendulum with respect to the vertical?

A) 30° B) 45° C) 60° D) 90° E) 75°

22. The amplitude of a damped oscillator becomes $\left(\frac{1}{3}\right)^{rd}$ in 2 seconds. If its amplitude after 6 seconds is $\frac{1}{n}$ times the original amplitude, the value of n is:

A) 3^2 B) $3\sqrt{2}$ C) $3\sqrt{3}$ D) 2^3 E) 3^3

23. Two sources of sound placed close to each other, are emitting progressive waves given by $y_1 = 4\sin 600\,\pi t$ and $y_2 = 5\sin 608\,\pi t$. An observer located near these two sources of sound will hear:

A) 8 beats per second with intensity ratio 81:1 between waxing and waning

B) 4 beats per second with intensity ratio 81:1 between waxin and waning

C) 4 beats per second with intensity ratio 25:16 between waxing and waning

D) 8 beats per second with intensity ratio 25:16 between waxing and waning

E) 8 beats per second with intensity ratio 1:81 between waxing and waning

24. The second overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe L meter long. The length of the open pipe will be:

A) 4L B) L C) 2L D) $\frac{L}{2}$ E) $\frac{L}{4}$

25. The radii of two speres are a and b respectively. They are at equal electric potential. The ratio of their surface density of charge is:

A) $\frac{a^2}{b^2}$ B) $\frac{b}{a}$ C) $\frac{a}{b}$ D) $\frac{b^2}{a^2}$ E) $\frac{b^3}{a^3}$

26. Two charges +q and –q are kept apart. Then, at any point on the right bisector of line joining the two charges:

A) the electric field strength is zero

B) the electric potential is zero

C) both electric potential and electric field strength are zero

D) both electric potential and electric field strength are non-zero

E) the electric field strength is non zero

27. Two condensers, one of capacity C and other of capacity C/2 are connected to a V volt battery, as shown in the figure. The work done in charging fully both the condensers is:

A) $\frac{1}{4}CV^2$ B) $\frac{3}{4}CV^2$ C) $\frac{1}{2}CV^2$ D) $2CV^2$ E) $3CV^2$

28. A conducting circular loop of radius r carries a constant current I. It is placed in a uniform magnetic field $\vec{B}$ such that $\vec{B}$ is perpendicular to the plane of the loop. The magneticd force acting on the loop is:

A) IrB B) $2\pi rIB$ C) zero D) πrIB E) None of these

29. A proton, a deuteron (nucleus of $_1H^2$) and an α – particle with the same KE enter in a region of uniform magnetic field, moving at right angles to B. What is the ratio of the radii of their circular paths?

A) $1:1:\sqrt{2}$ B) $1:\sqrt{2}:\sqrt{2}$ C) $\sqrt{2}:1:1$ D) $\sqrt{2}:\sqrt{2}:1$ E) $1:\sqrt{2}:1$

30. On connecting a battery to the two corners of a diagonal of a square conductor frame of side a the magnitude of the magnetic field at the centre will be:

A) zero B) $\mu_o / \pi a$ C) $2\mu_o / \pi a$ D) $4\mu_o / \pi a$ E) $8\mu_o / \pi a$

31. The period of oscillation of a magnetic needle in a magnetics field is 1.0 s. If the length of the needle is halved by cutting it, then the time period will be:

A) 1.0 sec B) 0.5 sec C) 0.25 sec D) 2.0 sec E) 0.75 sec

32. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon:

A) currents in the coils

B) materials of the wires of the coils

C) relative position and orientation of the coils

D) rates at which the currents are changing in the coils

E) Both B and D

33. The current through a coil of self-inductance L = 2mH is given by $I = t^2 e^{-t}$ at time t. How long it will take to make the emf zero?

A) 1 s B) 2s C) 3s D) 4s E) 5s

34. In non-resonant circuit, what will be the nature of the circuit for frequencies higher than the resonant frequency?

A) Resistive B) Capacitive C) Inductive D) Either A and B E) None of these

35. The resonant frequency of a circuit is f_0. If the capacitance is made 4 times the initial value, then the resonant frequency becomes:

A) $4f_0$ B) $2f_0$ C) f_0 D) $f_0 / 4$ E) $\frac{f_0}{2}$

36. An ideal transformer has 500 and 5000 turns in primary and secondary winding respectively. If the primary voltage is connected to a 6V battery, then the secondary voltage is :

A) 0 B) 60 V C) 0.6 V D) 6.0 V E) 1 V

37. A plane electromagnetic wave is incident on a material surface. If the wave delivers momentum p and energy E, then :

A) $p = 0, E = 0$ B) $p \neq 0, E \neq 0$ C) $p \neq 0, E = 0$ D) $p = 0, E \neq 0$ E) None of these

38. The frequency of a light wave in a material is 2×10^{14} Hz and wavelength is $5000\,\mathring{A}$. The refractive index of material will be:

A) 1.50 B) 3.00 C) 1.33 D) 1.40 E) 1.36

39. A convex lens of focal length f produces an image 1/n times that of the size of the object. The distance of the object from the lens is:

A) $\frac{f}{n}$ B) nf C) $(n-1)f$ D) $\frac{2f}{n}$ E) $(n+1)f$

40. If the ratio of amounts of scattering of two light waves is 1:4, the ratio of their wavelengths is:

A) $1:2$ B) $\sqrt{2}:1$ C) $1:\sqrt{2}$ D) $1:1$ E) $2:1$

41. Out of the following options which one can be used to produce a propagating electromagnetic wave?

A) A charge moving at constant velocity
B) A stationary charge
C) A chargeless particle
D) An accelerating charge
E) None of these

42. When a metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is $3V_0$ and when the same metallic surface is illuminated with a light of wavelength 2λ, the stopping potential is V_0. The threshold wavelength for the surface is:

A) 6λ B) 4λ C) $4\lambda/3$ D) 8λ E) 2λ

43. Electrons of mass m with de-Broglie wavelength (λ) fall on the target in an X-ray tube. The cut off wavelength (λ_0) of the emitted X-ray is:

A) $\lambda_0=\lambda$ B) $\lambda_0=\frac{2mc\lambda^2}{h}$ C) $\lambda_0=\frac{2h}{mc}$ D) $\lambda_0=\frac{2m^2c^2\lambda^2}{h^2}$ E) $\lambda_0=\frac{2m^2c\lambda^2}{h^2}$

44. The activity of a radioactive sample diminishes from 1024 to 128 in 2 minutes. In 6 minutes, the activity diminishes to:

A) 16 B) 8 C) 4 D) 2 E) 0

45. For a common emitter amplifier, the audio signal voltage across the collector resistance $2k\Omega$ is 2V. If the currect amplification factor of the transistor is 200 and the base resistance is $1.5k\Omega$, the input signal voltage and base current are:

A) 0.1 V and $1\mu A$
B) 0.15 V and $10\mu A$
C) 0.015 V and 1A
D) 0.0015 V and 1 mA
E) 0.0075 V and $5\mu A$

CHEMISTRY

46. Given $E^0_{Ag^+/Ag}=0.80\,V$ and $E^0_{Fe^{3+}/Fe^{2+}}=0.77\,V$. Calculate Δ_rG^0 for the reaction

$Fe^{2+}_{(aq)}+Ag^+_{(aq)}\rightarrow Fe^{3+}_{(aq)}+Ag(s)$

A) 28.95 kJ mol^{-1}
B) –2.895 kJ mol^{-1}
C) 289 kJ mol^{-1}
D) –5.78 kJmol^{-1}
E) 57.8 kJ mol^{-1}

47. If the solubility of silver chromate (Ag_2CrO_4)is S moles $litre^{-1}$, solubility product K_{sp} is

A) S^3 B) S^2 C) $2S^3$ D) $4S^3$ E) $27\ S^3$

48. If 30 ml 0.5 M H_2SO_4 is diluted to 500 ml. What is the molarity of the resulting solution?

A) 0.15 M B) 0.06 M C) 0.015 M D) 0.02 M E) 0.03 M

49. The molecule with zero dipole moment is

A) $SnCl_2$ B) NF_3 C) BF_3 D) CH_2Cl_2 E)H_2O

50. An outer orbital complex among the following is

A) $[Co(NH_3)_6]^{3+}$
B) $[Cr(NH_3)_6]^{3+}$
C) $[Fe(CN)_6]^{3-}$
D) $[MnCl_6]^{3-}$
E) $[Co(C_2O_4)_3]^{3-}$

51. The radius of the 2nd Bohr orbit of H atom is R. The radius of the 3rd orbit is

A) 3R B) 4R C) 4.4 R D) 9R E) 2.25 R

52. $SiCl_4$ is easily hydrolysed but CCl_4 is not. This is because

A) Bonding in CCl_4 is ionic

B) Silicon is less electronegative than carbon

C) Silicon can accept lone pair electrons from water molecules in d orbitals, but carbon cannot

D) Silicon can form hydrogen bonds but carbon cannot

E) Silicon is a metalloid

53. Among the following divalent cations, the one with highest magnetic moment is

A) Ni^{2+} B) Cr^{2+} C) Fe^{2+} D) Mn^{2+} E) Ni^{2+}

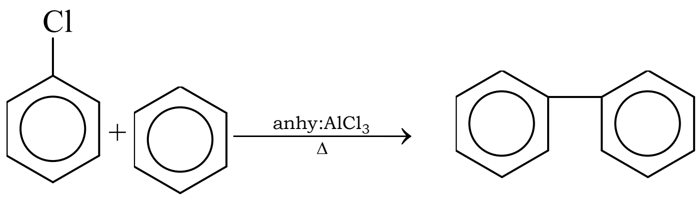
54. The volume of N_2 in mls at 0°C and 1 atm pressure that can be obtained by Dumas method from 1.2 g urea is

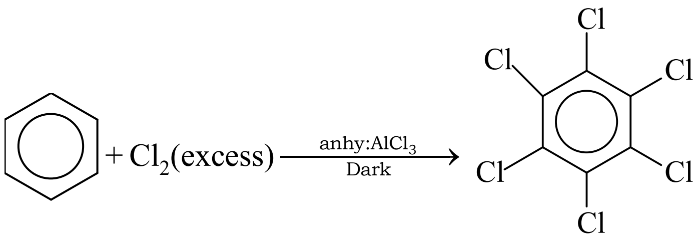
A) 224 ml B) 448 ml C) 112 ml D) 44.8 ml E) 336 ml1

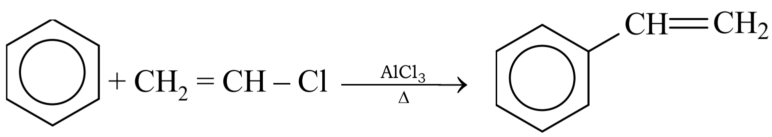
55. The H–C–C bond angle in ethylene is

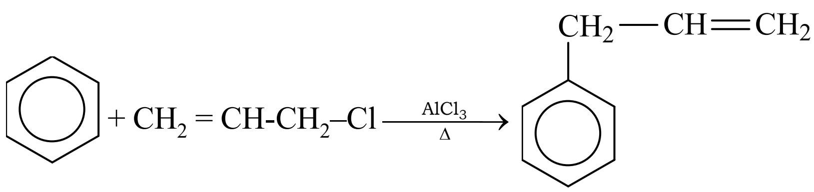
A) Exactly 120° B) 119.5° C) 121.7° D) 116.6° E) 109.5°

56. Which of the following reactions are possible?

1) 

2) 

3) 

4) 

A) 1, 2, 3 B) 2, 4 C) 1, 3, 4 D) 1, 3 E) 1, 2, 3, 4

57. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion (A) : The energy required to form Mg^{2+} from Mg is much higher than that required to produce Mg^{+}.

Reason (R): Mg^{2+} is small ion and carry more charge than Mg^{+}

In the light of the above statements choose the correct answer from the options given below

A) Both **A** and **R** are true but **R** is not the correct explanation of **A**

B) **A** is true but **R** is false

C) **A** is false but **R** is true

D) Both **A** and **R** are true and **R** is the correct explanation of **A**

E) Both **A** and **R** are false

58. In the equation $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$ the value of x is

A) 12 B) 2 C) 6 D) 14 E) 10

59. Match the List I with List II

	List I Natural aminoacid		List II One letter code
a)	Arginine	i)	D
b)	Aspartic acid	ii)	N
c)	Asparagine	iii)	A
d)	Alanine	iv)	R

Choose the correct answer from the options given below

A) (a) - (iv), (b)-(i), (c)- (iii), (d)- (ii) B) (a) - (i), (b)-(iii), (c)- (iv), (d)- (ii)

C) (a) - (iii), (b)-(i), (c)- (ii), (d)- (iv) D) (a) - (iv), (b)-(i), (c)- (ii), (d)- (iii)

E) (a) - (i), (b)-(iv), (c)- (iii), (d)- (ii)

60. Carbon forms two oxides CO & CO_2. This illustrates the law of:

A) definite proportions B) Multiple proportions

C) Reciprocal proportions D) Combining volumes E) Mass action

61. The frequency of a wave is $6 \times 10^{15}\ s^{-1}$. Its wave number would be

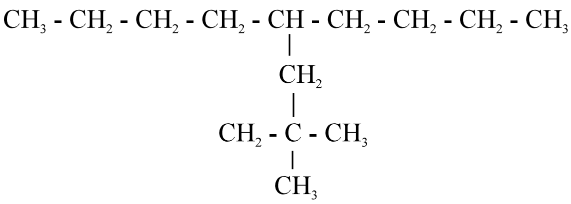
A) $10^5\ cm^{-1}$ B) $2\times10^{-5}cm^{-1}$ C) $2\times10^{-7}cm$ D) $2\times10^{7}cm^{-1}$ E) $2\times10^{5}cm^{-1}$

62. Identify the state function among the following

A) q B) q × w C) q/w D) q + w E) w

63. If the eqm. constant of the reaction $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$ at 750 K is 49, then the eqm. constant of the reaction $NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g)+\frac{3}{2}H_2(g)$ at the same temp is

A) $\frac{1}{49}$ B) 7 C) 49 D) $\frac{1}{7}$ E) 2401

64. Find the name of compound 

A) 5 – Neopentyldecane
B) 5 – (2, 2, – dimethyl propyl) nonane
C) 5 – tertbutylnonane
D) 5 - isopentylnonane
E) 5-(2- Ethylbutyl) nonane

65. Propene is more stable than ethene due to

A) Resonance
B) Hyperconjugation
C) Tautomerism
D) Inductive effect
E) Electromeric effect

66. Which of the following in dilute aq. solution has Vant Hoff factor = 4?

A) K_2SO_4 B) NaCl C) Urea D) Na_3PO_4 E) $Al_2(SO_4)_3$

67. If three faradays of electricity is passed through the solutions of $AgNO_3$, $CuSO_4$ and $AuCl_3$, the molar ratio of the cations deposited at the cathodes will be

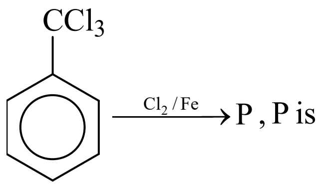
A) 1 : 1 : 1 B) 1 : 2 : 3 C) 3 : 2 : 1 D) 6 : 3 : 2 E) 6: 3 : 1

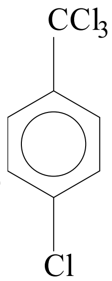
68. 99 % of a first order reaction was completed in 32 min. When will 99.9 % of the reaction complete ?

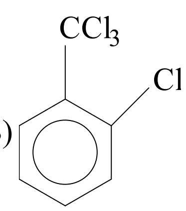
A) 50 min B) 46 min C) 49 min D) 48 min E) 64 min

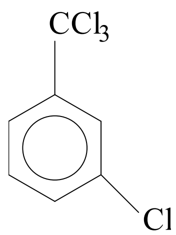
69. The isomerism present in $[Cr(NH_3)_5(CN)]Cl_2$ and $[Cr(NH_3)_5(NC)]Cl_2$ is:

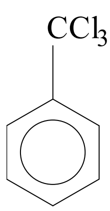
A) Geometrical isomerism
B) Position isomerism
C) Ionisation isomerism
D) Co-ordination isomerism
E) Linkage isomerism

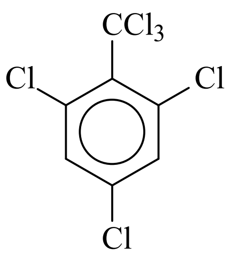
70. 

A) 

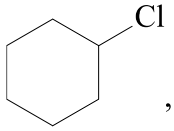
B) 

C) 

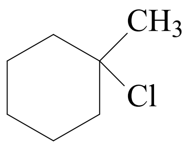
D) 

E) 

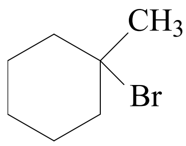
71. Predict the order of reactivity of the following compounds in SN^1 reactions.



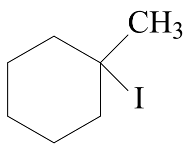
I



II



III



IV

A) I>II>III>IV B) IV>III>II>I C) III>II>IV>I D) IV>III>I>II E) II>IV>III>I

72. Arrange the following alcohols in the order of their boiling points :

1. CH_3OH

2. $CH_3 - CH_2 - CH_2 - CH_2 - OH$

3. $CH_3 - CH_2 - CHOH - CH_3$

4. $CH_3 - \underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}} - OH$

5. $CH_3 - CH_2 - CH_2 - CH_2 - CH_2 - OH$

A) 5 > 2 > 3 > 4 > 1 B) 5 > 2 > 4 > 3 > 1
C) 5 > 3 > 2 > 1 > 4 D) 3 > 5 > 1 > 4 > 2 E) 5 > 4 > 3 > 2 > 1

73. $CH_3 - CHO \xrightarrow{HCN} X \xrightarrow{H_2O/H^+} Y$. In the above sequence of reaction, Y is :

A) methanoic acid B) ethanoic acid
C) 2-hydroxypropanenitrile D) Prop-2-enoic acid
E) 2-hydroxypropanoic acid

74. The reaction, $RCH_2CH_2COOH \xrightarrow[Br_2]{Red\,P} R - CH_2 - \underset{\underset{Br}{|}}{CH} - COOH$ is called

A) Reimer-Tiemann reaction B) Hell-Volhard Zelinsky reaction
C) Cannizzaro reaction D) Sandmeyer reaction
E) Wolff-kishner reaction

75. How many 1^0 amines are possible for the formula $C_4H_{11}N$ (only structural isomers)

A) 1 B) 4 C) 3 D) 6 E) 5

MATHEMATICS

76. If $xy = \tan^{-1}(xy) + \cot^{-1}(xy)$, then $\frac{dy}{dx}$ is equal to

A) $\frac{y}{x}$ B) $\frac{-y}{x}$ C) $\frac{x}{y}$ D) $\frac{-x}{y}$ E) $\frac{1}{x}$

77. The value of k so that the point (7, k) lie on the lies passing through (3, 6) and (−5, 2) :

A) k = 8 B) k = −8 C) k = 10 D) k = 5 E) k = 3

78. If the lines $5x + py + 3 = 0$ and $10x - 14y - 9 = 0$ are parallel then p =

A) 7 B) 8 C) −7 D) 6 E) 3

79. The coordinates of one end point of the diameter of the circle $x^2 + y^2 + 8x + 6y = 43$ is (−2, −5). The other end point is :

A) (6, 11) B) (−6, 1) C) (−6, −1) D) (6, 10) E) (5, 1)

80. The circles whose equations are $x^2 + y^2 - 2ax + c^2 = 0$ and $x^2 + y^2 - 2by + c^2 = 0$ will touch externally if

A) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$ B) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ C) $a^2 + b^2 = c^2$ D) $a^2 - b^2 = c^2$ E) $a^2+b^2= 0$

81. The value of 'a' in order that f(x) = sin x – cos x – ax + b decreases for all real values of x is given by

A) $a \geq \sqrt{2}$ B) $a < \sqrt{2}$ C) $a \geq 1$ D) a<1 E) a>2

82. The minimum value of $2x^2+x-1$ is

A) $-1/4$ B) $3/2$ C) $-9/8$ D) $9/8$ E) $1/4$

83. If $f(x) = \begin{cases} 1 & \text{If } x \leq 3 \\ ax+b & \text{If } 3 < x < 5 \\ 7 & \text{If } 5 \leq x \end{cases}$ is continuous everywhere then

A) a = 3, b = 8 B) a = –3, b = –8 C) a = 5, b = –7
D) a = 7, b = –2 E) a = 3, b = –8

84. Two dice are thrown simultaneously. What is the probability of obtaining a total score of seven:

A) 1/3 B) 1/6 C) 1/9 D) 1/12 E) 1/2

85. A fair die is tossed once. The probability that either an even number or 3 will appear, is

A) 4/5 B) 3/4 C) 2/3 D) 3/5 E) 5/6

86. If the scalar projection of $\vec{a} = \lambda i + j + 4k$ on $\vec{b} = 2i + 6j + 3k$ is 4 units then λ is equal to :

A) 5 B) 3 C) 1 D) -5 E) 10

87. If $|\vec{a}| = 7, |\vec{b}| = 11$ $|\vec{a}+\vec{b}| = 10\sqrt{3}$ then $|\vec{a}-\vec{b}|$ equals

A) 10 B) $\sqrt{10}$ C) $2\sqrt{10}$ D) 20 E) 5

88. The direction cosines of the line joining the points (4, 3, -5) and (-2, 1, -8) are :

A) $\frac{6}{7},\frac{2}{7},\frac{3}{7}$ B) $\frac{2}{7},\frac{6}{7},\frac{3}{7}$ C) $\frac{5}{7},\frac{2}{7},\frac{3}{7}$ D) $\frac{2}{7},\frac{3}{7},\frac{5}{7}$ E) $\frac{3}{7},\frac{8}{7},\frac{9}{7}$

89. The equation $\frac{x^2}{2-r} + \frac{y^2}{r-5} + 1 = 0$ represents an ellipse if

A) $r > 2$ B) $r > 5$ C) $2 < r < 5$ D) $r < 2$ E) $r = 0$

90. Equation of directrix of the parabola $3y^2 = 20x$ is :

A) x – 5 = 0 B) 2x + 5 = 0 C) 3x + 5 = 0 D) 4x – 3 = 0 E) x+5 =0

91. The latus rectum of the ellipse $\frac{x^2}{1} + \frac{y^2}{4} = 1$ is :

A) 1 B) 2 C) $\sqrt{2}$ D) –1 E) -2

92. The value of the sum $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$, when $i = \sqrt{-1}$ equals

A) i B) i –1 C) –i D) 0 E) 1

93. If $\alpha + i\beta = \tan^{-1} z, z = x + iy$ and α is constant then the locus of z is

A) $x^2 + y^2 + 2x\cot 2\alpha = 1$ B) $\cot 2\alpha(x^2 + y^2) = 1 + x$

C) $x^2 + y^2 + 2y\tan 2\alpha = 1$ D) $x^2 + y^2 + 2x\sin 2\alpha = 1$

94. If $(x+iy)^{1/3} = a+ib$ then $\frac{x}{a}+\frac{y}{b}=$

A) $2(a^2-b^2)$ B) $4(a^2-b^2)$ C) $8(a^2-b^2)$ D) None

95. The real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number is

A) $n\pi+\pi/4$ B) $n\pi+(-1)^n\pi/4$ C) $2n\pi\pm\pi/2$ D) π E) $\pi/2$

96. The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3} is

A) reflexive but not symmetric B) reflexive but not transitive

C) symmetric and transitive D) Neither symmetric nor transitive E) not reflexive

97. If cos α is a root of $25x^2+5x-12=0, -1<x<0$. Then the value of $\sin 2\alpha$ is

A) $\frac{12}{25}$ B) $\frac{-12}{25}$ C) $\frac{-24}{25}$ D) $\frac{20}{25}$ E) $\frac{15}{25}$

98. If the mean of n observations $1^2, 2^2, 3^2, ...n^2$ is $\frac{46n}{11}$, then n is equal to :

A) 11 B) 12 C) 23 D) 22 E) 20

99. If $\begin{bmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ is non-invertible, then a =

A) 2 B) 1 C) 0 D) -1 E) -2

100. If $1^2+2^2+3^2+ ....+n^2 = 1015$, then the value of n is equal to:

A) 13 B) 14 C) 15 D) 17 E) 18

101. The order and degree of the differential eqn. $\left(\frac{d^3y}{dx^3}\right)^2+6\left(\frac{dy}{dx}\right)^3+\left(x^2-y^2\right)^4=0$ are:

A) 3,4 B) 3,3 C) 1,3 D) 3,2 E) 1,4

102. $\underset{x\to 0}{\text{Lt}} \frac{3\sin^2 x-2\sin x^2}{3x^2}=$

A) 0 B) 1 C) 1/3 D) 2/3 E) 4/3

103. The function $f(x)=x^3-3x$ is

A) increasing in $(-\infty,-1)\cup[1,\infty)$ and decreasing in (–1, 1)

B) decreasing in $(-\infty,-1]\cup[1,\infty)$ and increasing in $(-1,1)$

C) increasing in $(0,\infty)$ and decreasing in $(-\infty,0)$

D) decreasing in $(0,\infty)$ and increasing in $(-\infty,0)$ E) None of these

104. $\int \frac{dx}{4+5\sin x}=$

A) $\frac{1}{3}\log\left(\frac{2\tan x+1}{2\tan x+4}\right)+c$ B) $\frac{1}{3}\log\left(\frac{2\tan(x/2)+1}{2\tan(x/2)+4}\right)+c$

C) $\frac{1}{6}\log\left(\frac{2\tan(x/2)+1}{2\tan(x/2)+4}\right)+c$ D) $\frac{1}{3}\log\tan\frac{x}{2}+4x+c$ E) $\frac{1}{3}\log\left(\frac{2\tan(x/2)+1}{2\tan(x/2)+8}\right)+c$

105. The area bounded by parabola $y^2 = x$, straight line $y = 4$ and y axis is (in sq. units)

A) $\frac{16}{3}$ B) $\frac{64}{3}$ C) $7\sqrt{2}$ D) $\frac{7\sqrt{2}}{3}$ E) $\frac{\sqrt{3}}{2}$

106. The 21st and 22nd terms in the expansion of $(1 + x)^{44}$ are equal. Then value of x is:

A) 87 B) 78 C) 8/7 D) 7/8 E) 9/8

107. The term independent of x in the expansion of $\left(x^2 - \frac{1}{3x}\right)^9$ is equal to:

A) 58/213 B) 28/243 C) 243/28 D) 213/58 E) 88/213

108. If $A = \{x: x^2 - 5x + 6 = 0\}$, $B = \{2,4\}$ $C = \{4,5\}$ then $A \times (B \cap C)$ is:

A) $\{(2,4),(3,4)\}$ B) $\{(4,2),(4,3)\}$

C) $\{(2,4),(3,4),(4,4)\}$ D) $\{(2,2),(3,3),(4,4),(5,5)\}$ E) null set

109. The relation R defined in A = {1,2,3} by a Rb is $|a^2 - b^2| \le 5$. Which of the following is not true?

A) Domain of R = {1,2,3}

B) Range of R = {5}

C) $R^{-1} = R$

D) R = {(1,1), (2,2), (3,3), (2,1), (1,2), (2,3), (3,2)}

E) none

110. If $\sec\theta + \tan\theta = k$ then $\sin\theta =$

A) $\frac{k^2+1}{k^2-1}$ B) $\frac{k+1}{k-1}$ C) $\frac{k-1}{k+1}$ D) $\frac{k^2-1}{k^2+1}$ E) $\frac{1-k^2}{1+k^2}$

111. If $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$ where A is a non zero matrix then A =

A) $[-1 \ \ 2 \ \ -1]$ B) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ C) $[1 \ \ 2 \ \ 1]$ D) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ E) $[-1 \ \ 2 \ \ 1]$

112. If $x^2 - hx - 21 = 0$ and $x^2 - 3hx + 35 = 0$ $(h > 0)$ have a common root then h =

A) 3 B) 5 C) 2 D) 7 E) 1

113. $\tan\left[2\tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right)\right] =$

A) 4 B) 3 C) $\frac{1}{2}$ D) 5 E) 2

114. The solution of the differential equation $y\frac{dy}{dx} = x - 1$ satisfying $y(1) = 1$ is:

A) $y^2 = x^2 + 2x + 2$ B) $y^2 = 2x^2 - x - 1$ C) $y = 2x^2 - 2x + 1$

D) $y^2 = x^2 - 2x + 2$ E) $y = 2x^2 - 2x + 2$

115. If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$ then x =

A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) $\frac{\sqrt{3}}{2}$ E) $\frac{1}{\sqrt{3}}$

116. The range of $f(x) = \frac{x}{1+x^2}$ is given by

A) $\left[-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right)$ B) $\left[-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right]$

C) $(-\alpha,-1]\cup[1,+\alpha)$ D) $\left[-\frac{1}{2},\frac{1}{2}\right]$ E) $(-\alpha,\alpha)$

117. If $A=\begin{pmatrix}5 & 3\\ 14 & 11\end{pmatrix}$ then $13A^{-1}$ equals

A) A B) 2A C) adj A D) $\frac{A}{13}$ E) A^{-1}

118. If $x^2-8x+15>0$ and $x^2-11x+28\le 0$ then

A) $3<x<7$ B) $4<x<5$ C) $5\le x<7$ D) $5<x\le 7$ E) $5\le x\le 7$

119. If $\begin{vmatrix}x & x+y & x+y+z\\ 2x & 3x+2y & 4x+3y+2z\\ 3x & 6x+3y & 10x+6y+3z\end{vmatrix}=64$ then x =

A) 4 B) 2 C) 3 D) 6 E) 8

120. If $m\tan(\theta-30^\circ)=n\tan(\theta+120^\circ)$ then $\frac{m+n}{m-n}$ =

A) $2\cos 2\theta$ B) $\sin 2\theta$ C) $\tan 2\theta$ D) $2\sin 2\theta$ E) $2\tan 2\theta$

121. If $2\sin^2\theta = 3\cos\theta, 0\le\theta\le 2\pi$ then θ =

A) $\frac{\pi}{6},\frac{5\pi}{6}$ B) $\frac{\pi}{3},\frac{2\pi}{3}$ C) $\frac{\pi}{2},\pi$ D) $\frac{\pi}{3},\frac{5\pi}{3}$ E) $\frac{\pi}{4},\frac{5\pi}{4}$

122. $\int_0^{\pi/2}\sin x\sin 2x\,dx$ =

A) $\frac{1}{3}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{6}$ E) $\frac{2}{3}$

123. $\int_{-8}^{8}(ax^5+bx+c)dx$ is a function of ....

A) a, b B) c only C) b, c D) a, b, c E) a only

124. If $A=\begin{bmatrix}1 & 2 & -1\\ -1 & 1 & 2\\ 2 & -1 & 1\end{bmatrix}$ then $|\text{adj}(\text{adj}A)|$ =

A) 12^4 B) 13^4 C) 14^3 D) 14^4 E) 14^2

125. $\int \operatorname{cosec}^4 x dx$ is equal to

A) $\cot x - \frac{\cot^3 x}{3} + c$ B) $\tan x + \frac{\tan^3 x}{3} + c$

C) $-\cot x - \frac{\cot^3 x}{3} + c$ D) $-\tan x - \frac{\tan^3 x}{3} + c$ E) $\tan x - \frac{\tan^3 x}{3} + c$

126. $\int \sqrt{1 + \sin \frac{x}{2}} dx$ is equal to

A) $\frac{1}{4}\left[\cos \frac{x}{4} - \sin \frac{x}{4}\right] + c$ B) $4\left[\cos \frac{x}{4} - \sin \frac{x}{4}\right] + c$

C) $4\left[\sin \frac{x}{4} - \cos \frac{x}{4}\right] + c$ D) $4\left[\sin \frac{x}{4} + \cos \frac{x}{4}\right] + c$ E) $\frac{1}{4}\left[\cos \frac{x}{4} + \sin \frac{x}{4}\right] + c$

127. $\int_e^{e^2} \frac{dx}{x \log x}$

A) 1 B) log 1 C) log 2 D) -1 E) log 3

128. If the sum of 10 items is 12 and the sum of their squares is 18 then the standard deviation is

A) $\frac{3}{5}$ B) $\frac{2}{5}$ C) $\frac{1}{5}$ D) $\frac{4}{5}$ E) $\frac{\sqrt{3}}{5}$

129. $\int e^x (\tan x - \log \cos x) dx =$

A) $e^x \log(\operatorname{cosec} x) + c$ B) $e^x \log(\cos x) + c$

C) $e^x \log(\sec x) + c$ D) $e^x \log(\sin x) + c$ E) $e^x \log(\tan x) + c$

130. $\int_0^1 \frac{dx}{e^x + e^{-x}} =$

A) $\tan^{-1}\left(\frac{e-1}{e+2}\right)$ B) $\tan^{-1}\left(\frac{e+2}{e-1}\right)$ C) $\tan^{-1}\left(\frac{e+1}{e-1}\right)$ D) $\tan^{-1}(e)$ E) $\tan^{-1}\left(\frac{e-1}{e+1}\right)$

131. $\int_3^6 \frac{\sqrt{x} dx}{\sqrt{9-x} + \sqrt{x}} =$

A) 2 B) $\frac{3}{2}$ C) 1 D) $\frac{1}{2}$ E) $-\frac{3}{2}$

132. An AP consists of 23 terms. If the sum of the middle three terms is 141 and the sum of the last three terms is 261, then the first term is

A) 2 B) 3 C) 4 D) 5 E) 6

133. If $|\vec{a}| = 5$, $|\vec{b}| = 6$ and $\vec{a}.\vec{b} = -25$, then $|\vec{a} \times \vec{b}|$ is

A) $5\sqrt{11}$ B) $6\sqrt{11}$ C) $11\sqrt{5}$ D) $11\sqrt{6}$ E) 25

134. If two numbers p and q are chosen randomly with replacement from $\{1,2,3,4\}$, then the probability that $p^2 \ge 4q$ is

A) $\frac{3}{16}$ B) $\frac{5}{16}$ C) $\frac{7}{16}$ D) $\frac{9}{16}$ E) $\frac{11}{16}$

135. If the circles $(x+7)^2+(y-3)^2=36$ and $(x-5)^2+(y+2)^2=49$ touch each other externally, then the point of contact is

A) $\left(-\frac{19}{13},\frac{19}{13}\right)$ B) $\left(\frac{17}{13},\frac{9}{13}\right)$ C) $\left(-\frac{19}{13},\frac{9}{13}\right)$ D) $\left(\frac{-17}{13},\frac{9}{13}\right)$ E) $\left(\frac{19}{13},\frac{9}{13}\right)$

136. $\int (x+5)^4(x+4)(x+6)\,dx =$

A) $\frac{1}{30}(x+5)^4\left[5(x+5)^2-6\right]+C$ B) $\frac{1}{35}(x+5)^5\left[5(x+5)^2-7\right]+C$

C) $\frac{1}{42}(x+5)^6\left[6(x+5)-7\right]+C$ D) $\frac{1}{30}(x+5)^5\left[5(x+5)^2+6\right]+C$

E) $\frac{1}{35}(x+5)^5\left[5(x+5)^2+7\right]+C$

137. If $z=r(\cos\theta+i\sin\theta)$, then $\frac{z}{\bar{z}}+\frac{\bar{z}}{z}=$

A) $2\cos 2\theta$ B) $\cos 2\theta$ C) $2\cos\theta$ D) $2\sin\theta$ E) $2\sin 2\theta$

138. A die is rolled three times. The probability that the sum of the three number is 15 is

A) $\frac{5}{108}$ B) $\frac{5}{216}$ C) $\frac{11}{216}$ D) $\frac{6}{108}$ E) $\frac{7}{108}$

139. If $y=5^{\sin x}$, then $\frac{dy}{dx}$ at $x=\frac{\pi}{2}$ is

A) log 5 B) 2 log5 C) $\frac{1}{2}\log 5$ D) $\frac{1}{3}\log 5$ E) 0

140. If the length of the major axis of an ellipse is $\frac{17}{8}$ times that of the minor axis, the eccentricity is

A) $\frac{13}{17}$ B) $\frac{8}{17}$ C) $\frac{2\sqrt{2}}{17}$ D) $\frac{15}{17}$ E) $\frac{9}{17}$

141. Average weight of 35 students is 40 kgms. If the weight of the teacher is included, the average increases by $\frac{1}{2}$kgms. The weight of the teacher is

A) 52 B) 54 C) 55 D) 58 E) 60

142. The coefficient of x^{-9} in the expansion of $\left(\frac{x^2}{2}-\frac{2}{x}\right)^9$ is

A) 512 B) –512 C) 521 D) 251 E) 522

143. The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is

A) $\frac{1}{6}$ B) $\frac{1}{\sqrt{6}}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{3}$ E) 6

144. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to

A) $\frac{\log x}{1+\log x}$ B) $\frac{\log x}{1-\log x}$ C) $\frac{\log x}{(1+\log x)^2}$ D) $\frac{y\log x}{x(1+\log x)}$ E) $\frac{1+\log x}{\log x}$

145. The value of $\sin 12° \sin 48° \sin 54°$ is equal to

A) $\frac{2}{3}$ B) $\frac{1}{2}$ C) $\frac{1}{8}$ D) $\frac{1}{3}$ E) 3

146. The domain of $\sin^{-1}\left(\frac{2x+1}{3}\right)$ is

A) $(2,-1)$ B) $[-2,1]$ C) R D) $(-1,1)$ E) $(-2,0)$

147. For the matrix $A=\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 2 & 1 & 0\end{bmatrix}$, which is correct

A) $A^3+3A^2-I=0$ B) $A^3-3A^2-I=0$

C) $A^3+2A^2-I=0$ D) $A^3-A^2+I=0$ E) $A^3+A^2+I=0$

148. If $P=\begin{bmatrix}i & 0 & -i\\ 0 & -i & i\\ -i & i & 0\end{bmatrix}$ and $Q=\begin{bmatrix}-i & i\\ 0 & 0\\ i & -i\end{bmatrix}$ then PQ is equal to

A) $\begin{bmatrix}-2 & 2\\ 1 & -1\\ 1 & -1\end{bmatrix}$ B) $\begin{bmatrix}2 & -2\\ -1 & 1\\ -1 & 1\end{bmatrix}$ C) $\begin{bmatrix}2 & -2\\ -1 & 1\end{bmatrix}$ D) $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ E) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

149. The area of the triangle whose vertices are (1,2,3), (2,5,–1) and (–1,1,2) is

A) 150 sq. units B) 145 sq. units C) $\frac{\sqrt{155}}{2}$ sq.units D) $\frac{155}{2}$ sq.units E) $\frac{\sqrt{165}}{2}$ sq.units

150. If $y = x\tan y$, then $\frac{dy}{dx}$ is equal to

A) $\frac{\tan y}{x-x^2-y^2}$ B) $\frac{y}{x-x^2-y^2}$ C) $\frac{\tan y}{y-x}$ D) $\frac{\tan x}{x-y^2}$ E) $\frac{\tan y}{x+x^2+y^2}$

PHYSICS

1. B Mobility, $\mu = \frac{\text{Drift velocity } (v_d)}{\text{Electric field } (E)}$

$\therefore \ [\mu] = \frac{[v_d]}{[E]} = \frac{[M^0LT^{-1}]}{[MLT^{-3}A^{-1}]} = [M^{-1}T^2A]$

2. B Time taken by coin to reach the floor is given by:

$h = 1/2gt^2 \qquad (\because u = 0)$

or $t = \sqrt{\frac{2h}{g}}$ In stationary lift, $t_1 = \sqrt{\frac{2h}{g}}$

In upward moving lift with constant acceleration a, $g' = g + a$

$\therefore \qquad t_2 = \sqrt{\frac{2h}{(g+a)}}$

Clearly, $g' > g$ Thus, $t_2 < t_1$.

3. B If $\vec{C}$ is resultant of $\vec{A}$ and $\vec{B}$, then

$|\vec{C}| = \sqrt{A^2 + B^2 + 2AB\cos 120°}$

$|\vec{C}| = \sqrt{A^2 + B^2 - AB} \qquad \left[\text{As } \cos 120° = -\frac{1}{2}\right]$

Similarly, $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos 120°}$

$= \sqrt{A^2 + B^2 + AB} \qquad |\vec{A} - \vec{B}| > C$

4. A $\tan\theta = \frac{u\sin\theta}{u\cos\theta} = \frac{2}{1}$

The desired equation is, $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$= x \times 2 - \frac{10x^2}{2(\sqrt{2^2+1^2})^2\left(\frac{1}{\sqrt{5}}\right)^2}$ or $y = 2x - 5x^2$.

5. A Now, $a = \frac{\sqrt{F_1^2 + F_2^2}}{m} = \frac{F_3}{m} = \frac{R_3}{m}$

6. E

7. A

8. B According to law of conservation of momentum, $mv + 0 = (m + M)V$

or $V = \left(\frac{mv}{m+M}\right)$

As the block rises to height h, hence $V = \sqrt{2gh}$

or $\frac{m\upsilon}{(m+M)} = \sqrt{2gh}$ $\therefore$ $\upsilon = \frac{(M+m)}{m}\cdot\sqrt{2gh}$

9. E Angular momentum, $L = I\omega$ Rotational kinetic energy, $K = \frac{1}{2}I\omega^2$

$\frac{L}{K} = \frac{I\omega}{\frac{1}{2}I\omega^2} = \frac{2}{\omega}$ or $L = \frac{2K}{\omega}$

$\therefore$ $\frac{L'}{L} = \frac{K'}{K}\times\frac{\omega}{\omega'} = \left(\frac{K}{2K}\right)\left(\frac{\omega}{2\omega}\right)$ $L' = \frac{L}{4}$

10. D Mass of disc $\propto$ area

$\therefore$ $M_A = 4M_B$ (as $R_A = 2R_B$)

$\therefore$ $\frac{I_A}{I_B} = \frac{\frac{1}{2}M_A R_A^2}{\frac{1}{2}M_B R_B^2} = 4\times 4 = 16.$

11. A

12. C Force at a height $(3L/4)$ from its lower end

$=$ weight suspended + weight of $\frac{3}{4}$ of the chain

$= W_1 + \frac{3W}{4}$ $\therefore$ Stress $= \frac{W_1 + \frac{3}{4}W}{S}$.

13. E Excess pressure inside a soap bubble $P = \frac{4T}{R}$,

Where T is the surface tension of the soap solution, R is the radius of a soap bubble.

$\therefore$ $\frac{P_1}{P_2} = \frac{R_2}{R_1}$ or $\frac{3P_2}{P_2} = \frac{R_2}{R_1}$ or $\frac{R_1}{R_2} = \frac{1}{3}$

$\therefore$ $\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

14. C The wett ability of a surface by a liquid depends primarily on angle of contact between the surface and the liquid.

15. B Let the volume of the ball be V. Force on the ball due to upthrust = Vdg

Net upward force = $Vdg - VDg$

$\therefore$ Upward acceleration is given by: $VDa = Vdg - VDg$

$\therefore \quad a = \left(\frac{d-D}{D}\right)g$

Velocity on reaching the surface, $v = \sqrt{2ah}$

Further $v = \sqrt{2gH}$ $\therefore$ $2ah = 2gH$

or $H = \frac{ah}{g} = \left(\frac{d-D}{D}\right)h = \left(\frac{d}{D} - 1\right)h.$

16. C Stream-line motion is more likely for liquids having high viscosity and low density.

17. A $\frac{C}{100} = \frac{F-32}{180}$ $\frac{\theta}{100} = \frac{2\theta-32}{180}$ or $\theta = 160°.$

18. E $\frac{m_A C_A}{m_B C_B} = \frac{(4/3)\pi r_A^3 \rho_A C_A}{(4/3)\pi r_B^3 \rho_B C_B}$ $= \left(\frac{r_A}{r_B}\right)^3 \frac{\rho_A C_A}{\rho_B C_B} = \left(\frac{1}{2}\right)^3 \times \left(\frac{2}{1}\right) \times \left(\frac{1}{3}\right) = \frac{1}{12}.$

19. C $\Delta T = \Delta T_0 e^{-\lambda t}$

$T = 2T e^{-\lambda(10\,\text{min})}$

$\therefore \quad \Delta T = 2T e^{-\lambda(20\,\text{min})} = 2T\left(\frac{1}{2}\right)^2 = \frac{T}{2}$ So, $T_f = T + \frac{T}{2} = \frac{3T}{2}$

20. D **Given :** $P \propto T^3$

or $PT^{-3} = K$ ...(i)

For adiabatic process:

$PV^{\gamma} = \text{constant } (C)$

or $P\left(\frac{RT}{P}\right)^{\gamma} = C$

or $P^{1-\gamma}T^{-\gamma} = C'$

or $PT^{\gamma/1-\gamma} = C'' = K$ ...(ii)

Comparing eqn. (i) and (ii), we get; $\frac{\gamma}{1-\gamma} = -3$ or $\gamma = 3/2.$

21. D $T = 3mg$

$$T - mg = \frac{mv^2}{l} = 2mg \qquad v = \sqrt{2gl}$$

$$\frac{1}{2}mv^2 = mgl(1-\cos\theta) \qquad \text{or} \qquad \theta = 90^\circ.$$

22. E Amplitude of a damped oscillator at any instant t is given by

$A = A_0 e^{-bt/2m}$

where A_0 is the original amplitude. When $t = 2$ s, $A = \frac{A_0}{3}$

$$\therefore \quad \frac{A_0}{3} = A_0 e^{-2b/2m} \Rightarrow \frac{1}{3} = e^{-b/m} \qquad \ldots(i)$$

When $t = 6$ s, $A = \frac{A_0}{n}$

$$\therefore \quad \frac{A_0}{n} = A_0 e^{-6b/2m}$$

$$\frac{1}{h} = e^{-3b/m = (e^{-b/m})^3}\ \frac{1}{n} = \left(\frac{1}{3}\right)^3 \quad \text{[Using eqn. (i)]} \qquad \therefore \quad n = 3^3.$$

23. B $\omega_1 = 600\pi$ or $n_1 = \frac{600\pi}{2\pi} = 300\ s^{-1}$ $\qquad \omega_2 = 608\pi$ or $n_2 = \frac{608\pi}{2\pi} = 304\ s^{-1}$

$\therefore$ Number of beats $= n_2 - n_1 = 304 - 300 = 4\ s^{-1}$

$$\text{Intensity ratio} = \frac{I_{max}}{I_{min}} = \left(\frac{a_2 + a_1}{a_2 - a_1}\right)^2 \qquad = \left(\frac{5+4}{5-4}\right)^2 = \frac{81}{1}.$$

24. C At resonance, first overtone of closed pipe = second overtone of open pipe

$$3 \times \frac{v}{4L_1} = \frac{3}{2L_2}v \qquad \text{or} \qquad \frac{1}{4L_1} = \frac{1}{2L_2}$$

$$\frac{L_1}{L_2} = \frac{2}{4} = 1:2$$

As, $L_2 = L$, hence $L_2 = 2L$

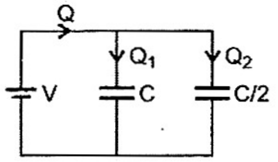
25. B $V_1 = V_2$

$$\frac{q_1}{4\pi\varepsilon_0 a} = \frac{q_2}{4\pi\varepsilon_0 b}, \quad \frac{q_1}{q_2} = \frac{a}{b}$$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi a^2}{q_2/4\pi b^2} = \frac{q_1}{q_2} \times \frac{b^2}{a^2} = \frac{a}{b} \times \frac{b^2}{a^2} = \frac{b}{a}.$$

26. B At equatorial point, $E_e = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$ (directed from $+q$ to $-q$) and $V_e = 0$.

27. B As the capacitors are connected in parallel, therefore, potential difference across both the condensers remains the same.



$\therefore \quad Q_1 = CV \; ; \; Q_2 = \frac{C}{2}V$

Also, $Q = Q_1 + Q_2 = CV + \frac{C}{2}V = \frac{3}{2}CV$

Work done in charging fully both the condensers is given by :

$W = \frac{1}{2}QV = \frac{1}{2} \times \left(\frac{3}{2}CV\right)V = \frac{3}{4}CV^2$

28. C The forces acting on various small current carrying elements of the circumference of the loop will be distributed randomly in all possible directions. The vector addition of such randomly distributed forces will be zero.

29. E $qvB = \frac{mv^2}{r}$ $\therefore$ $r = \frac{mv}{qB} = \sqrt{\frac{2mE}{q^2B^2}}$

where, E = KE of particle.

$r_p = \sqrt{\frac{2mE}{e^2B^2}}, \quad r_d = \sqrt{\frac{2 \times 2m \times E}{e^2B^2}}$ and $r_\alpha = \sqrt{\frac{2 \times 4m \times E}{(2e)^2 B^2}}$

$\therefore \quad r_p : r_d : r_\alpha = 1 : \sqrt{2} : 1.$

30. A

31. B We know that, $T = 2\pi\sqrt{\frac{I}{MB_H}}$ ... (i)

When length is halved, M is also halved and I becomes (I/8).

$\therefore \quad T' = 2\pi\sqrt{\frac{(I/8)}{(M/2)B_H}} = \frac{T}{2} = \frac{1.0}{2} = 0.5\,\text{sec}$

32. C We know that mutual inductance of the pair of coils depends upon the geometry of the coils, distance between the coils, relative position and orientation of the coils, number of turns in the coils, permeability of the medium in the coils and degree of coupling, i.e., extent to which the magnetic flux due to primary current links with the secondary coil. Therefore, option C is correct.

33. B $I = t^2e^{-t}$ $\therefore \quad \frac{dI}{dt} = 2te^{-t} - t^2e^{-t} = te^{-t}(2-t)$

The induced emf is

$\varepsilon = -L\frac{dI}{dt}$

According to given problem, $\varepsilon = 0$

or $\frac{dI}{dt} = 0$ (Since, $L \neq 0$)

or $e^{-t}t(2-t) = 0$

either $t = 0$ or $t = 2s$

$t = 2s$ matches with the option B.

34. C At resonant frequency, $X_L = X_C \left(\omega t = \frac{1}{\omega C}\right)$

At frequencies higher than resonance frequencies, $X_L > X_C$

i.e., behaviour is inductive.

35. E In L-C-R series circuit, resonance frequency f_0 is given by :

$L\omega = \frac{1}{C\omega}$ or $\omega^2 = \frac{1}{LC}$

$\therefore \quad \omega = \sqrt{\frac{1}{LC}} = 2\pi f_0$

$\therefore \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$ or $f_0 \propto \frac{1}{\sqrt{C}}$

36. A Transformers working with DC will have zero output.

37. B An electromagnetic wave has both momentum and energy.

38. B $\mu = \frac{\text{velocity of light in vacuum } (c)}{\text{velocity of light in medium } (v)}$

$\therefore \quad v = \nu\lambda = 2 \times 10^{14} \times 5000 \times 10^{-10}$

In the medium, $v = 10^8$ m/s

$\therefore \quad \mu = \frac{v_{\text{vacuum}}}{v_{\text{medium}}} = \frac{3 \times 10^8}{10^8} = 3$

39. E $m = \frac{v}{u} = \frac{1}{n}$ or $v = \frac{u}{n}$

Here, v is +ve and u is −ve.

$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or $\quad \frac{n}{u} - \frac{1}{(-u)} = \frac{1}{f}$

or $\quad u = (n+1) f.$

40. B From Rayleigh scattering concept, $I \propto \frac{1}{\lambda^4}$ or $\frac{I_1}{I_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$

or $\left(\frac{\lambda_2}{\lambda_1}\right)^4 = \frac{1}{4}$ or $\frac{\lambda_2}{\lambda_1} = \left(\frac{1}{2}\right)^{1/2}$ or $\frac{\lambda_1}{\lambda_2} = \frac{\sqrt{2}}{1}.$

41. D An accelerated charge is used to produce oscillating electric and magnetic field, hence the electromagnetic wave.

42. B Einstein's photoelectric equation:

$\frac{hc}{\lambda} = e(3V_0) + W$ ...(i) and $\frac{hc}{2\lambda} = e(V_0) + W$ ...(ii)

Solving these equations, we get;

$\frac{hc}{\lambda} = \frac{3hc}{2\lambda} + W - 3W$ or $2W = \frac{hc}{2\lambda}$ or $\frac{hc}{\lambda_0} = \frac{hc}{4\lambda}$

So, $\lambda_0 = 4\lambda.$

43. B K.E. of electrons $= \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{h^2}{2m\lambda^2}$

So maximum energy of photon will also be this much.

$\therefore \frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2}$ or $\lambda_0 = \frac{2mc\lambda^2}{h}$

44. D $\frac{128}{1024} = \frac{1}{8} = \left(\frac{1}{2}\right)^3$

3 half-life periods = 2 minute 6 minute means 9 half-life periods.

$\therefore \quad N = N_0 \left(\frac{1}{2}\right)^9$ $= 1024\left(\frac{1}{512}\right) = 2.$

45. C Here, $V_o = 2$ V

$R_C = 2$ kΩ, $R_B = 1.5$ kΩ $\beta = 200$

Voltage gain, $A_V = \frac{V_o}{V_i} = \beta \frac{R_C}{R_B}$ or $V_i = \frac{V_o R_B}{\beta R_C}$

Substituting the values, we get;

$V_i = \frac{2\ V \times 1.5\ k\Omega}{200\ V \times 2\ k\Omega} = 0.0075\ V$ $I_C = \frac{V_o}{R_C} = \frac{2\ V}{2\ k\Omega} = 1\ mA$

Also, $\beta = \frac{I_C}{I_B}$

or $I_B = \frac{I_C}{200} = \frac{1 \times 10^{-3}\ A}{200}$

$I_B = 0.5 \times 10^{-5}$ A. $= 5 \times 10^{-6}$ A $= 5\ \mu$A

CHEMISTRY

46. B $E^0_{cell} = E^0_{Ag^+/Ag} - E^0_{Fe^{3+}/Fe^{2+}} = 0.80 - 0.77 = 0.03$ V

$\Delta_r G^0 = -nFE^0 = -1 \times 96500 \times 0.03 = -965 \times 3\ J\,mol^{-1} = -2895\ J\ mol^{-1}$

$= -2.895\ kJ\ mol^{-1}$

47. D $Ag_2CrO_4 \longrightarrow 2Ag^+ + CrO_4^{2-}$

S 2S S

$K_{sp} = (2S)^2 \times S = 4S^3$

48. E 1000 ml 0.5 M H_2SO_4 is 0.5 mole H_2SO_4

$\therefore$ 30 ml contain $\frac{0.5 \times 30}{1000} = 0.015$ mole

Vol. of solution = 500 ml = 0.5 litre

$\therefore$ Molarity $= \frac{0.015}{0.5} = 0.03$

49. C BF_3 zero dipole moment

50. D $[MnCl_6]^{3-}$ is the outer orbital complex (sp^3d^2 hybridisation)

51. E If the radius of the first orbit of H atom is R_1. Radius of the second orbit R_2 is $4R_1$ (but $4R_1 = R$).

Radius of third orbit, R_3 is $9R_1 = 9 \times \frac{R}{4} = 2.25$ R

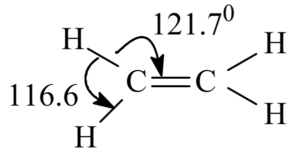
52. C Silicon can expand its octet beyond '8' because of d orbitals i.e. it can extend its coordination number beyond 4

53. D Mn^{2+} maximum number of unpaired electrons

54. B % of Nitrogen $= \frac{V_0}{8 \times W}$ where V_0 = Vol. of N_2 in mls at NTP

W = Mass of substance in grams

$\therefore \frac{28 \times 100}{60} = \frac{140}{3}$; $\therefore \frac{140}{3} = \frac{V_0}{8 \times 1.2}$ or $V_0 = \frac{140 \times 8 \times 1.2}{3} = 448$ ml

55. C 

56. B Only 2 and 4 reactions

57. D $Mg \rightarrow Mg^{+}$ only IE_1

$Mg \rightarrow Mg^{2+}$ $IE_1 + IE_2$ & $IE_2 > IE_1$

58. E The balanced equation is

$2IO_3^- + 10I^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

59. D Arginine-R, Aspartic acid - D, Asparagine - N, Alanine - A

60. B

61. E $\overline{v} = \frac{v}{c} = \frac{6\times10^{15}s^{-1}}{3\times10^{10}cm\,s^{-1}} = 2\times10^{5}cm^{-1}$

62. D

63. D

64. B

65. B

66. D

67. D $Ag^{+} + e^{-} \longrightarrow Ag$; 3F of electricity will produce 3 moles of Ag. $Cu^{2+} + 2e^{-} \longrightarrow Cu$.

3F of electricity will produce 3/2 moles of Cu

$Au^{3+} + 3e^{-} \longrightarrow Au$.

3F of electricity will produce 1 mole of Au $\therefore$ molar ratio of

Ag : Cu : Au

3 : 3/2 : 1

6 : 3 : 2

68. D $k = \frac{2.303}{32}\log\frac{a}{a-0.99a} = \frac{2.303}{32}\log 10^{2}$

$= \frac{2.303}{16}\min^{-1}$

$t_{99.9\%} = \frac{2.303}{K}\log\frac{a}{a-0.999_a}$

$= \frac{2.103}{K}\log 10^{3} = \frac{3\times2.303\times16}{2.303} = 48\min$

69. E

70. C CCl_3 is m-directing.

71. B The first compound is a 2^0 alkyl halide while all others are 3^0.

Since 3^0 is more reactive than 2^0 further reactivity increases in the order Cl<Br<I

72. A

73. E

74. B

75. B

MATHEMATICS

76. B

77 A Eqn. of line through (3, 6) & (−5, 2) is $y-6=\frac{2-6}{-5-3}(x-3)$

$\Rightarrow x-2y+9=0$; (7, k) lies on this line ; $\therefore$ $7-2k+9=0$; $k=8$

78. C $m_1=m_2 \Rightarrow \frac{-5}{p}=\frac{5}{7} \Rightarrow p=-7$

79. C Centre = (−4, −3), Let (α, β) be the other end ; $\frac{-2+\alpha}{2}=-4 \Rightarrow \alpha=-6; \frac{-5+\beta}{2}=-3 \Rightarrow \beta=-11$;

Other end point = (−6, −1)

80. B $x^2+y^2-2ax+c^2=0$; $x^2+y^2-2by+c^2=0$; centre $c_1=(a, 0)$, centre $c_2=(0, b)$

radius $r_1=\sqrt{a^2-c^2}$; radius $r_2=\sqrt{b^2-c^2}$; the two circles touch externally then $c_1c_2=r_1+r_2$

$\sqrt{a^2+b^2}=\sqrt{a^2-c^2}+\sqrt{b^2-c^2}$; $a^2b^2=c^2(a^2+b^2)$; $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

81. A $f'(x)=\cos x+\sin x-a$; $f'(x)<0$; $\Rightarrow a>\cos x+\sin x$

since $-\sqrt{2}\le \cos x+\sin x\le\sqrt{2}$ $\therefore a\ge\sqrt{2}$

82. C $y'=4x+1$, put $y'=0$ $\Rightarrow x=\frac{-1}{4}$

$y''=4=+ve$; y is minimum at $x={}^{-1}\!/_4$; Minimum value = $2\left({}^{-1}\!/_4\right)^2+\left({}^{-1}\!/_4\right)-1={}^{-9}\!/_8$

83. E

84. B (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1); Required probability = 6/36 = 1/6

85. C In this case the favourable number of cases, is 4 because getting 2, 4, 6 and 3 are favourable events.

86. A

87. C $|a+b|^2+|\vec{a}-b|^2=2\left(|a|^2+|b|^2\right)$; $|\vec{a}-\vec{b}|=2\sqrt{10}$

88. A

89. C Equation of the ellipse $\frac{x^2}{r-2}+\frac{y^2}{5-r}=1$

$\Rightarrow r-2>0 \;\&\; 5-r>0 \Rightarrow 2<r<5$

90. C $3y^2=20x$; $y^2=\frac{20}{3}x$; $4a=\frac{20}{3}$; $a=\frac{5}{3}$

Eqn. of directrix $x+a=0$; $x+(5/3)=0$; $3x+5=0$

91. A $a^2=2$; $b^2=1$; L.R. $=\frac{2b^2}{a}=\frac{2}{2}=1$

92. B $\sum_{n=1}^{13}\left(i^n+i^{n+1}\right)=\sum_{n=1}^{13}i^n(1+i)$

93. A Since $\alpha+i\beta=\tan^{-1}z=\tan^{-1}(x+iy)$; $\alpha-i\beta=\tan^{-1}(x+iy)$; $2\alpha=\tan^{-1}(x+iy)+\tan^{-1}(x-iy)$

94. B $(x+iy)^{1/3}=a+ib$ cubing $x+iy=(a+ib)^3$

95. C

96. A

97. C Solving we get $\cos\alpha=\frac{-4}{5}$;$\therefore=3/5$; Now $\sin 2\alpha=2\sin\alpha\cos\alpha$

98. A

99. B Given matrix is non invertible if $\begin{vmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{vmatrix} = 0$

ie, if 1 (2 - 5) - a (1 - 10) + 2 (1 - 4) = 0=> a = 1

100. B

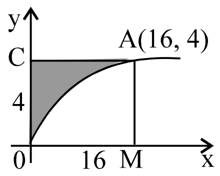
101. D Degree of $\frac{d^3y}{dx^3}$ is 2

102. C

103. A $f'(x) = 3x^2 - 3 = 3(x^2 - 1) \geq 0 \Rightarrow x \in (-\infty, -1] \cup [-1, \infty)$

and $f'(x) \leq 0 \Rightarrow x^2 \leq 1 \Rightarrow (x-1)(x+1) \leq 0 \Rightarrow -1 < x < 1 \Rightarrow x \in (-1, 1)$

104. B

105. B 

Area $= 4 \times 16 - \int_0^{16} \sqrt{x}\,dx$

106. D

107. B

108. A $B \cap C = \{4\}, A = \{2, 3\}; A \times (B \cap C) = \{(2, 4), (3, 4)\}$

109. B

110. D

111. E $(3 \times 1)(1 \times 3) = (3 \times 3) \Rightarrow$ A is $1 \times 3 \Rightarrow A = [p\ \ q\ \ r]$

112. D Common root $\alpha \Rightarrow \alpha^2 - h\alpha - 21 = 0$ and $\alpha^2 - 3h\alpha + 35 = 0$. Subtracting : $h\alpha = 28$

$\Rightarrow \alpha^2 = 49 \Rightarrow \alpha = 7$

113. E $\tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right) = \theta \Rightarrow E = \tan 2\theta$

114. D $y\frac{dy}{dx} = x - 1;\ ydy = (x-1)dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} - x + c;\ y(1) = 1 \Rightarrow c = 1;\ \therefore y^2 = x^2 - 2x + 2$

115. D $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ and $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

116. D Given $y = \frac{x}{1+x^2}$;$x^2y - x + y = 0$; $B^2 - 4AC \geq 0$ for $x \leftarrow R$; $1 - 4y^2 \geq 0$ $4y^2 \leq 1$; $y \leftarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$

117. C $A^{-1} = \frac{adj\,A}{|A|}$, here $|A| = 13$

118. D $(x-3)(x-5) > 0$ and $(x-4)(x-7) \leq 0$

119. A $R_2 \to R_2 - 2R_1;\ R_3 \to R_3 - 3R_1$

120. A $\frac{m}{n} = \frac{\sin(\theta + 120^\circ)}{\cos(\theta + 120^\circ)} \times \frac{\cos(\theta - 30^\circ)}{\sin(\theta - 30^\circ)};\ \frac{m+n}{m-n} = \frac{\sin(\theta + 120^\circ + \theta - 30^\circ)}{\sin(\theta + 120^\circ - \theta + 20^\circ)}$

121. D $2\cos^2\theta + 3\cos\theta - 2 = 0$

122. E $I = \int_0^{\pi/2} \sin x \times 2\sin x \cos x\, dx, \sin x = t$

123. B $ax^5 + bx$ odd function

124. D $\text{adj}(\text{adj } A) = |A|^{n-2} A = 14A$

125. C $I = \int \text{cosec}^2 x\left(1+\cot^2 x\right)dx$

126. C $I = \int \sqrt{\sin^2\frac{x}{4}+\cos^2\frac{x}{4}+2\sin\frac{x}{4}\cos\frac{x}{4}}\,dx \; ; = \int\left(\sin\frac{x}{4}+\cos\frac{x}{4}\right)dx$

127. C $\int_e^{e^2}\frac{1/x\,dx}{\log x} = \left[\log(\log x)\right]_e^{e^2} = \log 2$

128. A $\sigma^2 = \frac{1}{n}\Sigma x^2 - \left(\frac{1}{x}\Sigma x\right)^2$

129. C $I = \int e^x\left[f(x)+f'(x)\right]dx$ where $f(x) = \log(\sec x)$

130. E $I = \int_0^1 \frac{e^x dx}{e^{2x}+1}$ put $e^x = u$

131. B Replace $x \to a+b-x$

132. B
$a_{11} + a_{12} + a_{15} = 141$ &
$a_{21} + a_{22} + a_{25} = 261$.
Subtracting, $30d = 120$
$d = 4$.
$3a + 33d = 141$

133. A $|a \times b|^2 + (a \cdot b)^2 = |a|^2 |b|^2$

134. C
Total number of selections $= 4 \times 4 = 16$
$p^2 \geq 4q \Rightarrow (p,q) = \{(2,1),(3,1)(3,2),(4,1),(4,2),(4,3)(4,4)\}$

135. C

136. B
$x+5 = t$
$I = \int t^4(t^2-1)dt$

137. A
$z = re^{i\theta}, \; \bar{z} = re^{-i\theta}$
$\frac{z}{\bar{z}} + \frac{\bar{z}}{z} = e^{i2\theta} + e^{-i2\theta} = 2\cos 2\theta$

138. A

Total number of triplets = 216

Favourable results are (3,6,6), (4,6,5) (4,5,6), (5,6,4), (5,5,5), (5,4,6), (6,6,3), (6,5,4), (6,4,5), (6,3,6)

139. E

140. D

$a = \frac{17}{8}b.$

$c^2 = a^2e^2 = a^2 - b^2 = \frac{289}{64}b^2 - b^2$

$e^2 = \frac{225b^2}{64a^2} \Rightarrow e = \frac{15}{8}\frac{b}{a} = \frac{15}{8}\frac{8}{17}$

141. D

142. B

Let the coefficient of x^{-9} is in the $(r+1)$th term in the expansion of $\left(\frac{x^2}{2}-\frac{2}{x}\right)^9$, then

$$T_{r+1} = {}^9C_r\left(\frac{x^2}{2}\right)^{9-r}\cdot\left(-\frac{2}{x}\right)^r$$

$$= {}^9C_r\frac{x^{18-2r}}{2^{9-r}}\cdot\frac{(-1)^r\cdot 2^r}{x^r}$$

$$= {}^9C_r\frac{x^{18-3r}}{2^{9-2r}}(-1)^r$$

$\therefore \quad x^{18-3r} = x^{-9}$

$\Rightarrow \quad 18 - 3r = -9$

$\Rightarrow \quad 27 = 3r \Rightarrow r = 9$

$\therefore$ Coefficient of $x^{-9} = {}^9C_9\frac{1}{2^{-9}}(-1)^9$

$= -2^9 = -512$

143. B

The given lines are

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 2-1 & 4-2 & 5-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1$$

and $\sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2}$

$= \sqrt{(15-16)^2 + (12-10)^2 + (8-9)^2}$

$= \sqrt{6}$

$\therefore$ Required shortest distance

$$= \frac{\begin{vmatrix} l_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2}}$$

$= \frac{1}{\sqrt{6}}$

144. C $y \log x = x - y$

$$y(1+\log x) = x \; ; \; y = \frac{x}{1+\log x} \; ; \; \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

145. C $\sin 12° \sin 48° \sin 54°$

$$= \frac{1}{2}[\cos 36° - \cos 60°] \sin 54°$$

$$= \frac{1}{2}\left[\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right]\left[\frac{\sqrt{5}+1}{4}\right]$$

$$= \frac{1}{2}\left[\frac{\sqrt{5}-1}{4}\right]\left[\frac{\sqrt{5}+1}{4}\right] = \frac{5-1}{2 \times 4 \times 4}$$

$$= \frac{1}{8}$$

146. B Let $y = \sin^{-1}\left(\frac{2x+1}{3}\right)$

$\therefore \quad -1 \le \frac{2x+1}{3} \le 1$

$\Rightarrow \quad -3 \le 2x+1 \le 3$

$\Rightarrow \quad -4 \le 2x \le 2$

$\Rightarrow \quad -2 \le x \le 1$

$\therefore \quad x \in [-2, 1]$

147. B

$$A^2 = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 6 & 2 \\ 3 & 4 & 1 \end{bmatrix}, A^3 = A^2 . A = \begin{bmatrix} 7 & 9 & 3 \\ 15 & 19 & 6 \\ 9 & 12 & 4 \end{bmatrix}$$

Hence $A^3 - 3A^2 - I = 0$

148. B

$$PQ = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}\begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2 - i^2 & i^2 + i^2 \\ 0 + 0 + i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$$

149. C The area of triangle whose vertices are (1, 2, 3), (2, 5, –1) and (–1, 1, 2) is $\frac{\sqrt{155}}{2}$ sq. units.

150. B $y = x \tan y$

$\therefore \quad \frac{dy}{dx} = x . \sec^2 y \frac{dy}{dx} + \tan y$

$$(1 - x \sec^2 y)\frac{dy}{dx} = \tan y$$

$$\left\{1 - x\left(1 + \frac{y^2}{x^2}\right)\right\}\frac{dy}{dx} = \tan y$$

$$(x - x^2 - y^2)\frac{dy}{dx} = x \tan y$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x - x^2 - y^2}$